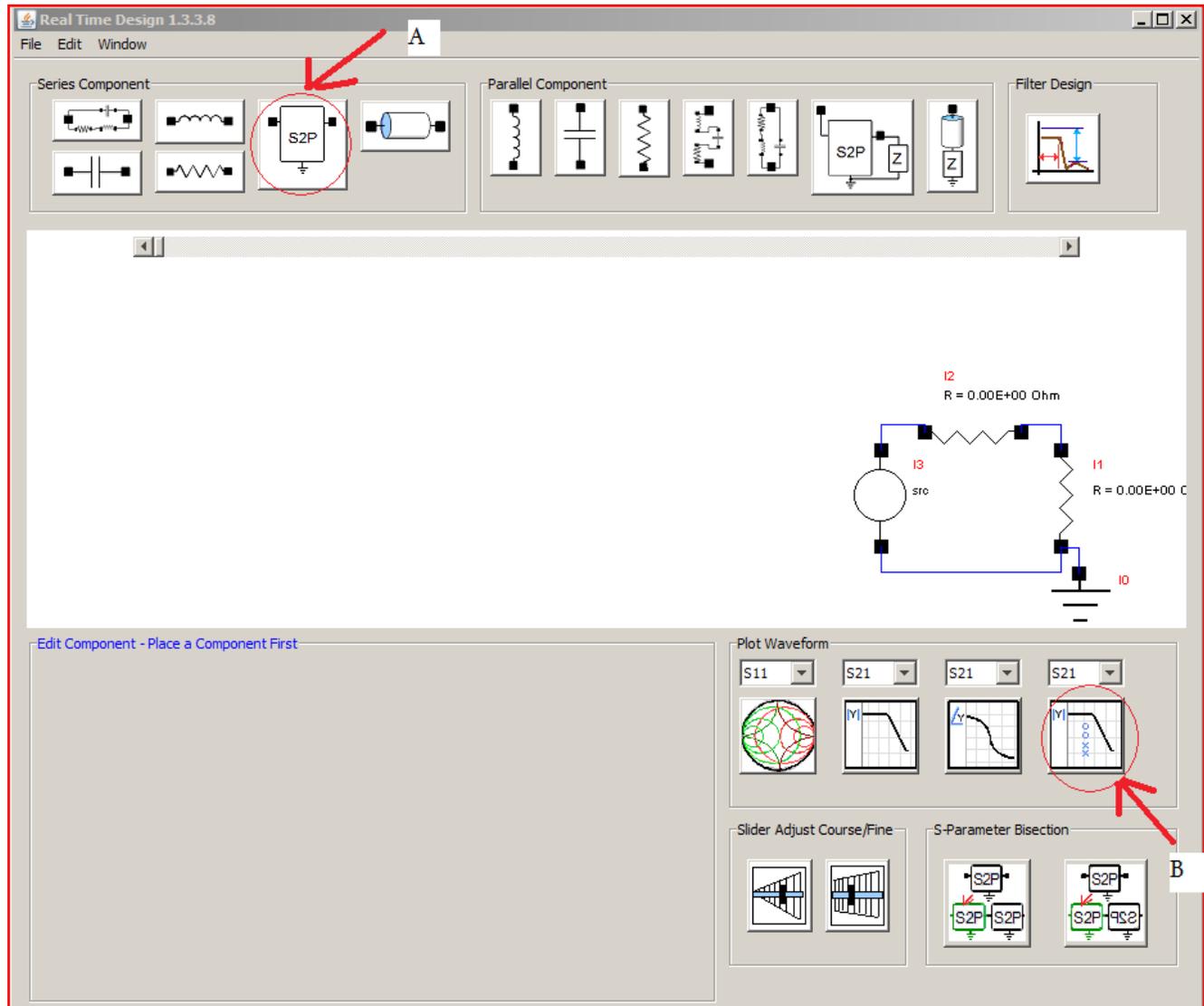


# Complex Polynomial Fitting and Root Removal/Discovery

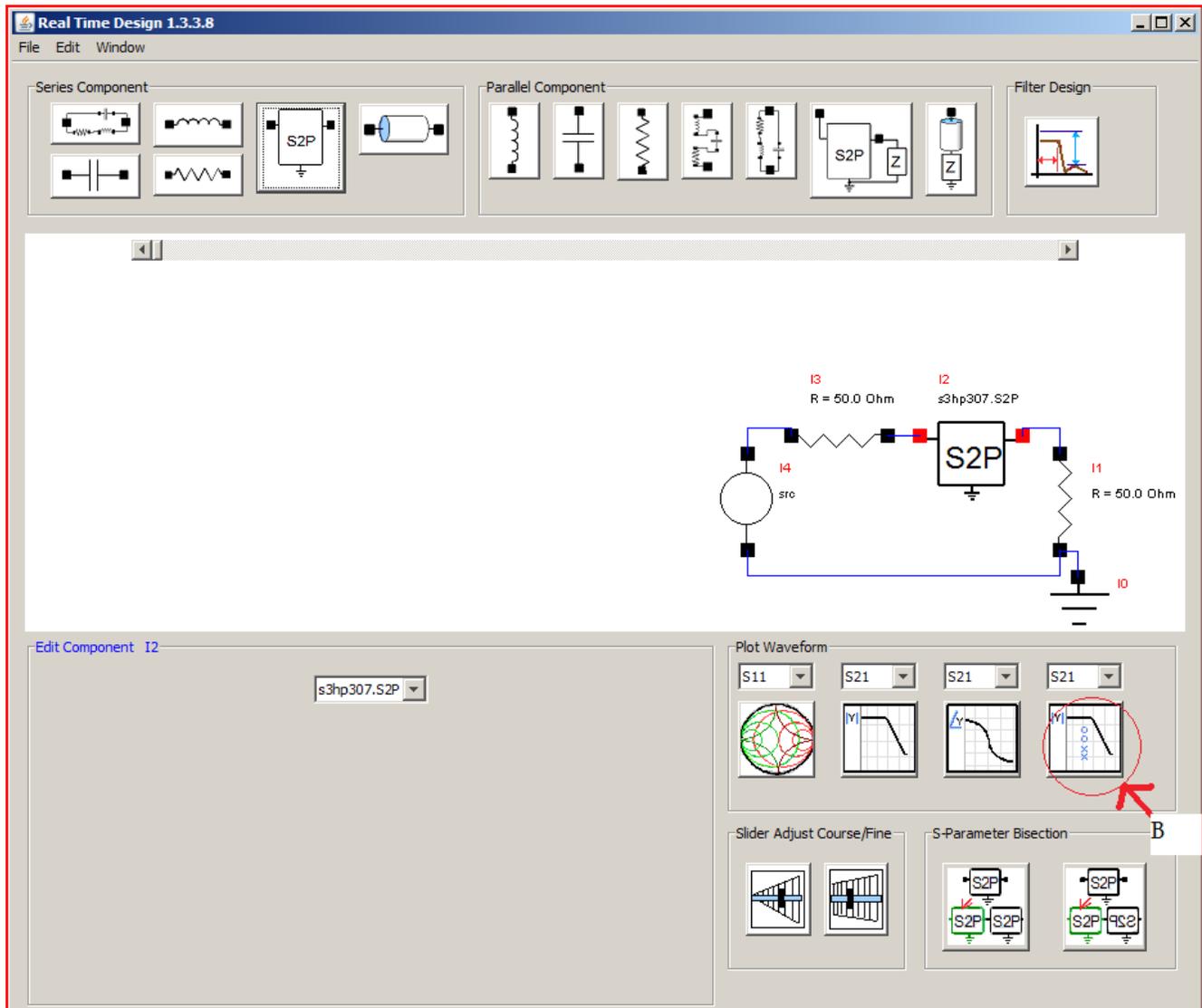
Example Polynomial Fitting to S21 of measured s-parameters.

1. Load a S2P file by selecting the S2P series component labeled “A” in the following image.

Note: Returning users will notice the addition of new button labeled “B” in the following image. This will be used to initiate the polynomial fitting and pole zero discovery feature.



2. After inserting the component, as in the image shown below, select the fitting button to initiate the fitting and pole zero discovery feature. labeled “B” in the following image



Fitting is performed primarily using the Rational Fractional Polynomial similar to the method described in the paper

“PARAMETER ESTIMATION FROM FREQUENCY RESPONSE MEASUREMENTS

USING RATIONAL FRACTION POLYNOMIALS”, Mark H. Richardson & David L. Formenti, Presented at 1st IMAC Conference, Orlando, FL. November, 1982

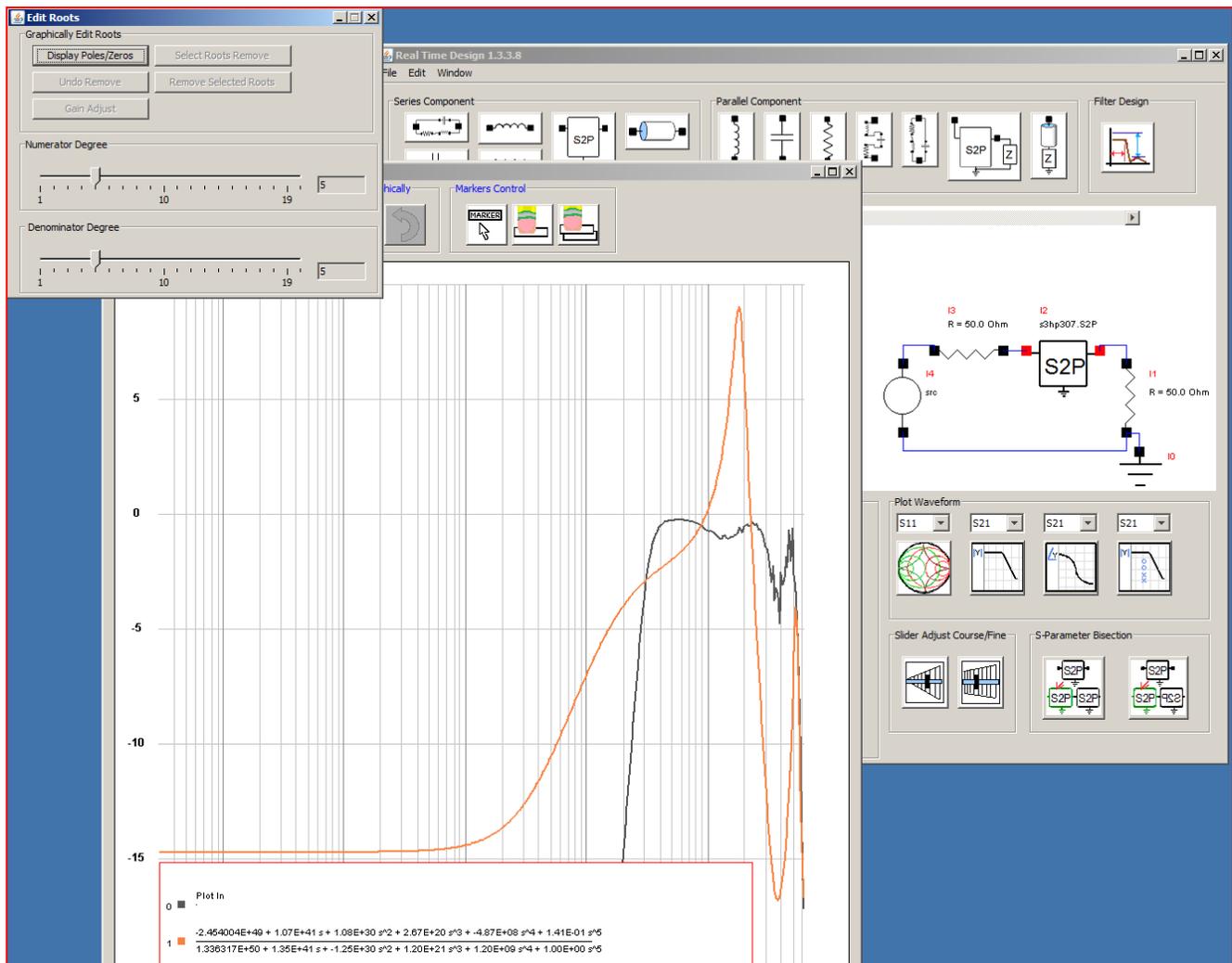
While this algorithm is good at approximating a complex fit polynomial to noise added data it is not sufficiently insensitive to noisy data without extreme over determination or a lack of meaningful convergence, thus this implementation of that algorithm is preceded first by determining which s-parameter entries violate the passivity criterion and thus excluding these entries. This reduces the likelihood of noise exceeding or dominating the fitting of meaningful measurement information.

Over-determination can usually be corrected as will be shown further on in this presentation.

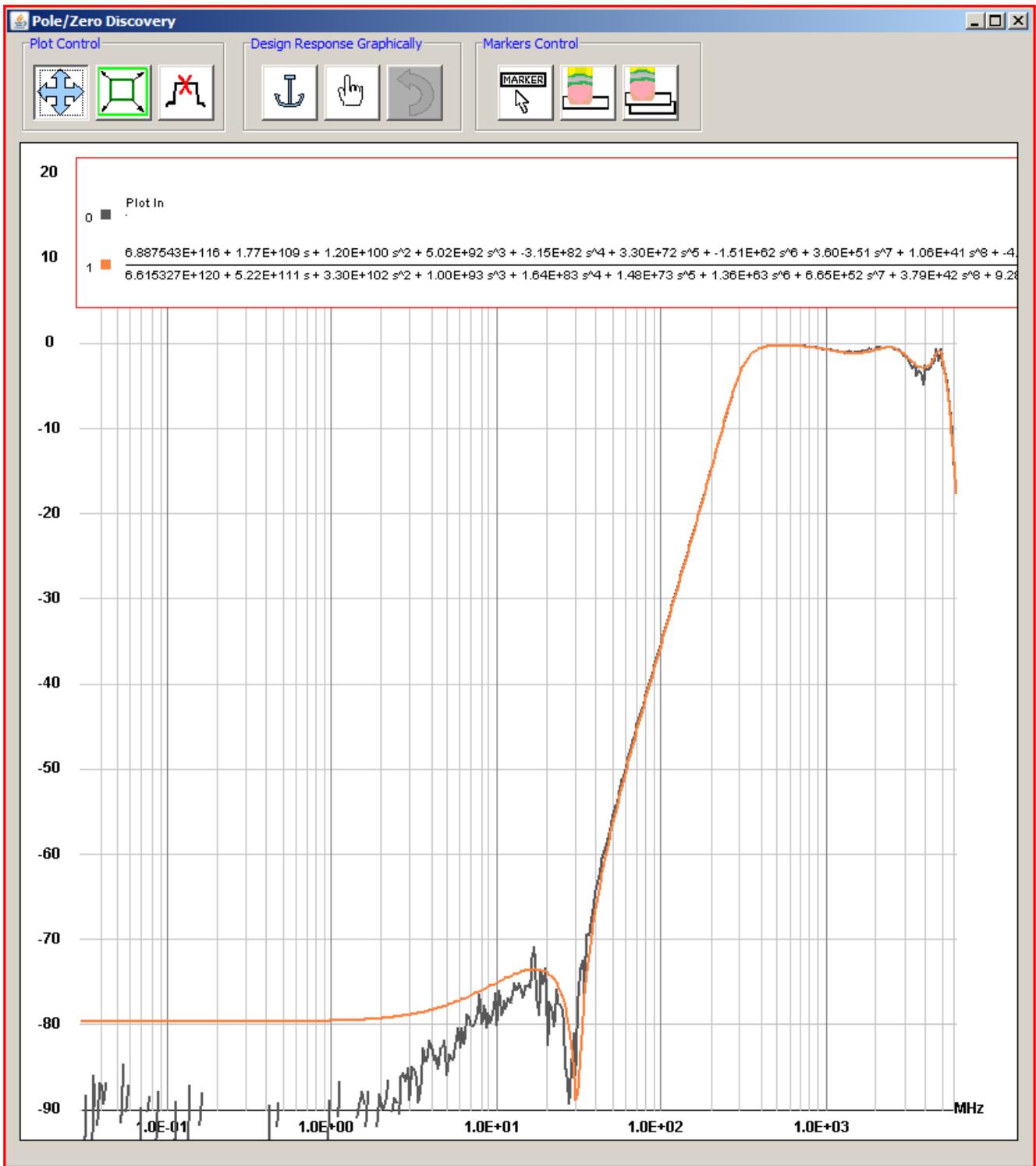
## Fitting

The following windows will open.

3. By adjusting the numerator and denominator degree slider the complex polynomial will best fit the data shown in the plot. The polynomial will be displayed in the panel shown below inside the plot window.

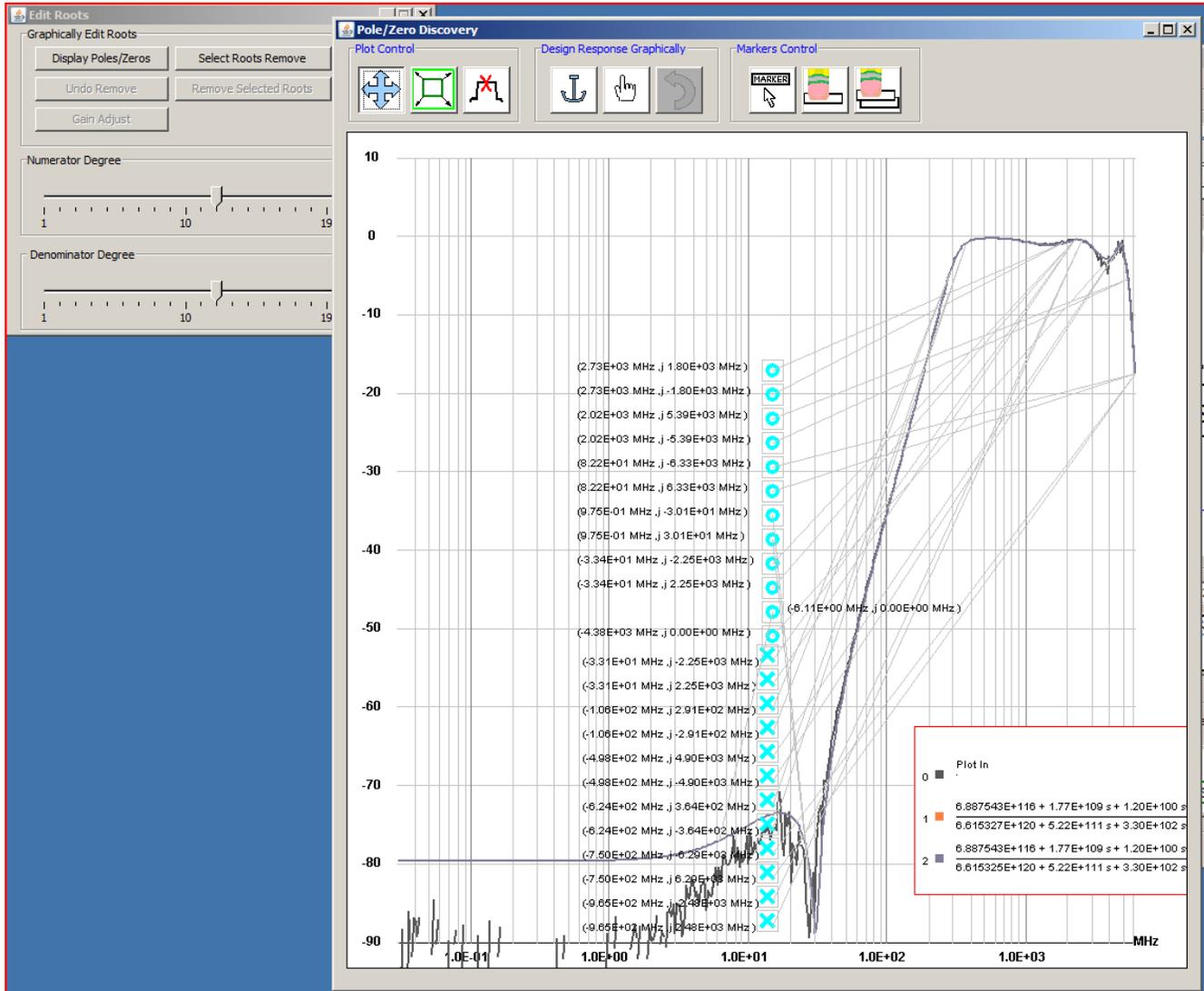


After finding the right degree (or least wrong) of both the numerator and denominator. Below is the best fit polynomial plotted next to the original measurement data.



At this point the poles and zeros can be displayed.

4. By pressing the “Display Poles/Zeros” button the poles and zeros will appear in the plot window. Also note that the panel in this window will show a third line with a polynomial derived by the displayed poles and zeros.

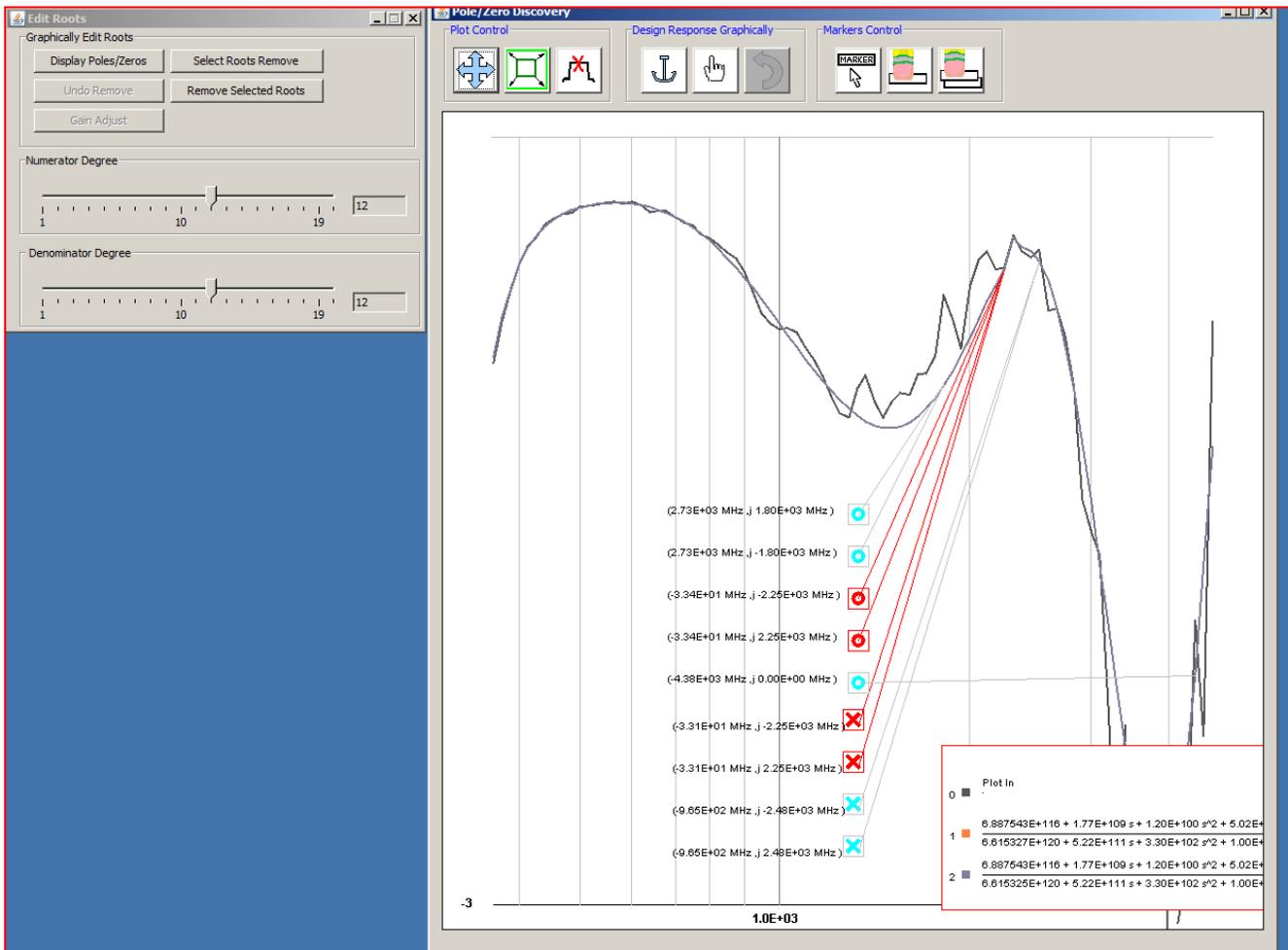


The user can now reduce the over determination from the fitting process.

## Pole Zero Removal

Selecting a complex pole and/or zero will highlight both of the complex pairs and if this root is complex both will be displayed and only both can be removed. Typically a product of the fitting algorithm over determination is a complex set of poles and set of zeros will lie at similar complex frequencies and likely be right on top of one another. As in the zoomed in part of the pass band shown below.

5. Press the “Select Roots Remove” button then highlight the roots to be removed by clicking on one, (if the root is complex then both conjugate roots will highlight).

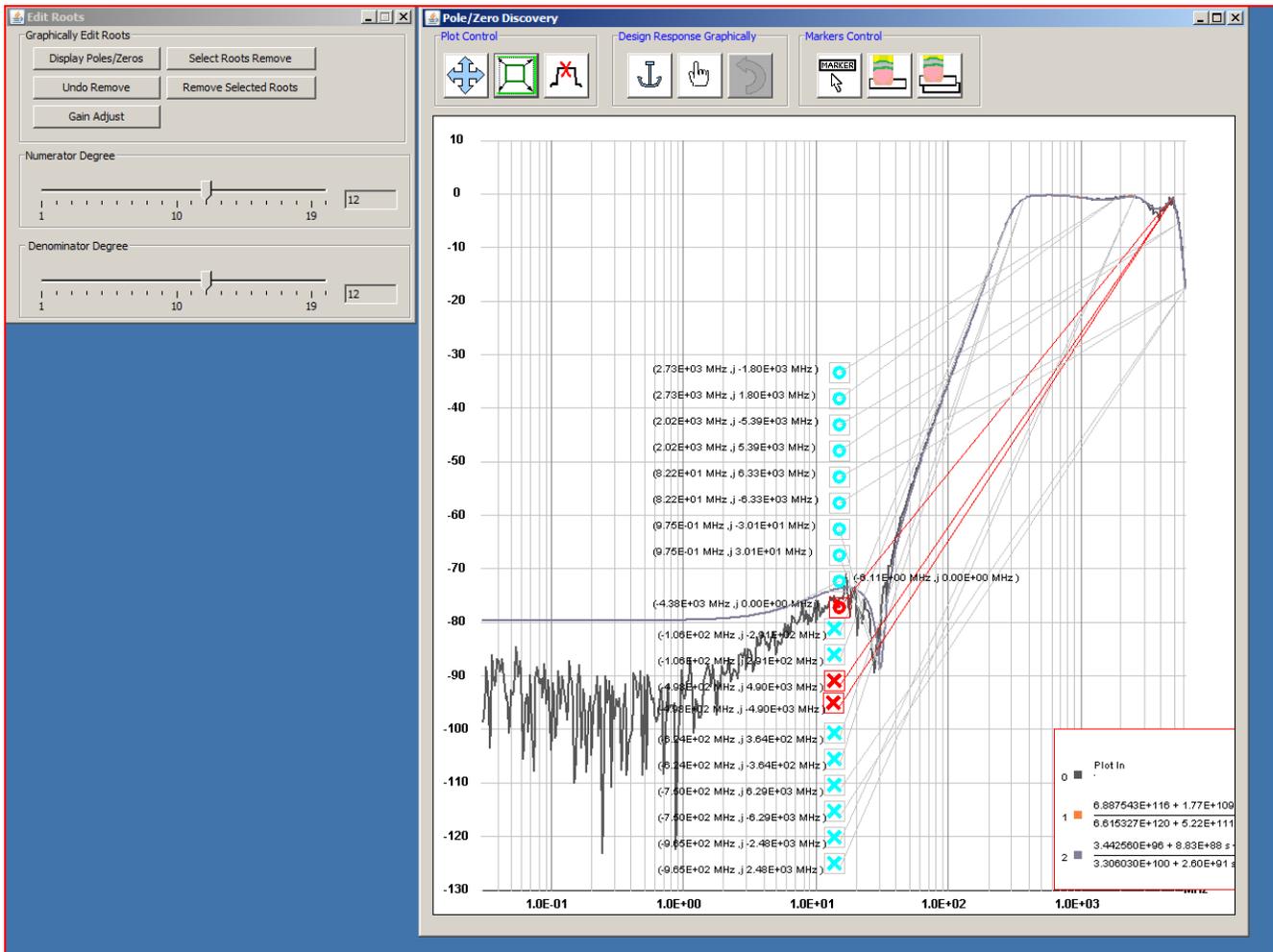


6. Select “Remove Selected Roots” and the roots will be removed and the polynomial recalculated with the remaining roots. As shown below. Note that the order of the root generated polynomial has reduced by 2 degrees when compared to the original fit shown on line two of the display panel.



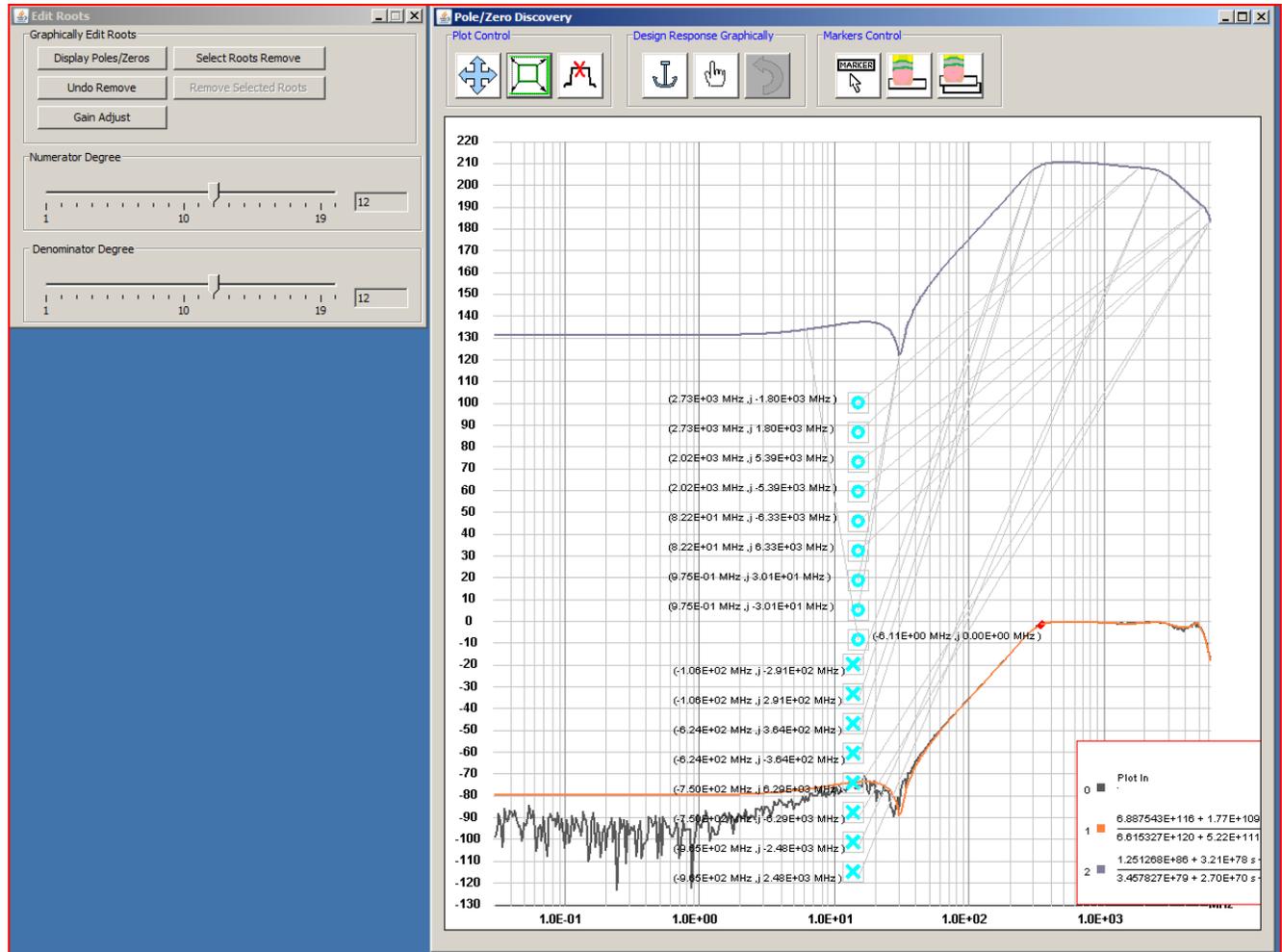
## Gain normalization after nonequivalent pole/zero removal

Depending on the proximity of the pole/zero to be removed the resulting polynomial may be quite different in terms of gain. The next example shows what happens when a real Zero and a complex pole pair is selected and removed and the resulting gain error as a result of this is remedied.



After removing the resulting plot shows the large discrepancy between the new polynomial and the original fit.

7. By mouse selecting a point on the original fit plot below represented as the orange plot. (The point displayed red highlighted dot on the orange plot). After choosing the point of gain normalization press the “Gain Adjust” button to normalize. Hint: pick a point sufficiently far from the removed points to



obtain the best approximate.

Below is the result of three rounds of pole/zero removal and after correcting the gain. The majority of the waveform is preserved while the order of the polynomial has been reduced significantly. If the root removal results are undesired then press “Undo remove” button to return to state prior to last removal.

**Edit Roots**

Graphically Edit Roots

Display Poles/Zeros    Select Roots Remove

Undo Remove    Remove Selected Roots

Gain Adjust

Numerator Degree

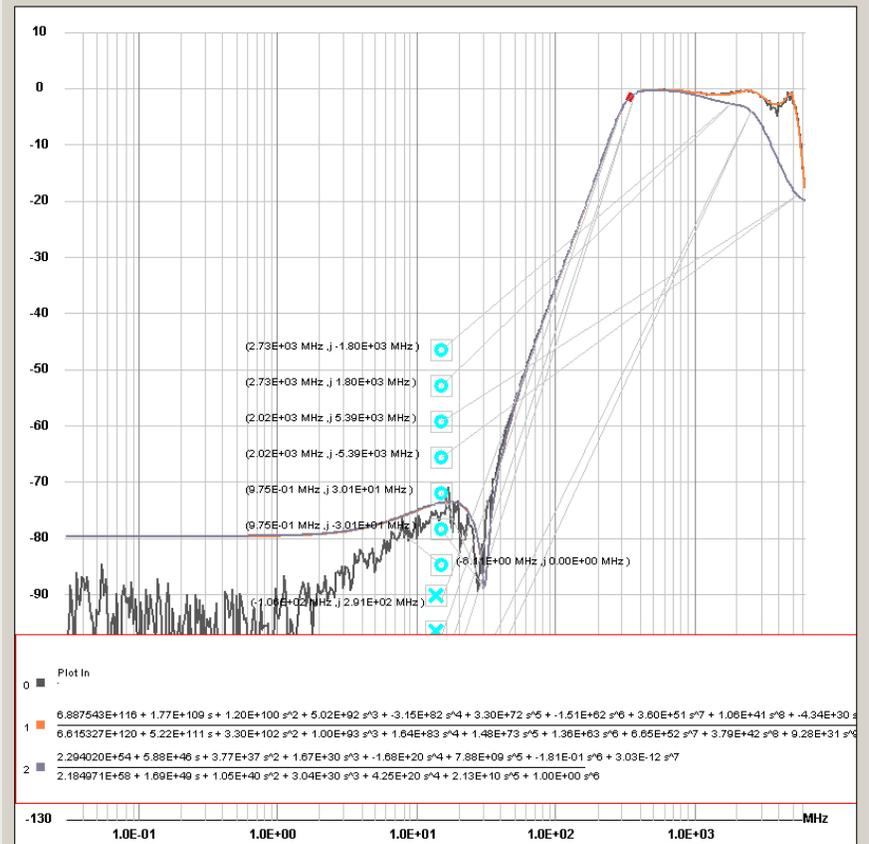
1    10    19    12

Denominator Degree

1    10    19    12

**Pole/Zero Discovery**

Plot Control    Design Response Graphically    Markers Control



## Real-time observation of poles and zeros during design

In the following example circuit which was previously synthesized using the filter synthesis feature which can be invoked by pressing the button in the “Filter Design” category of the main widow. The example below is the default bandpass elliptical filter.

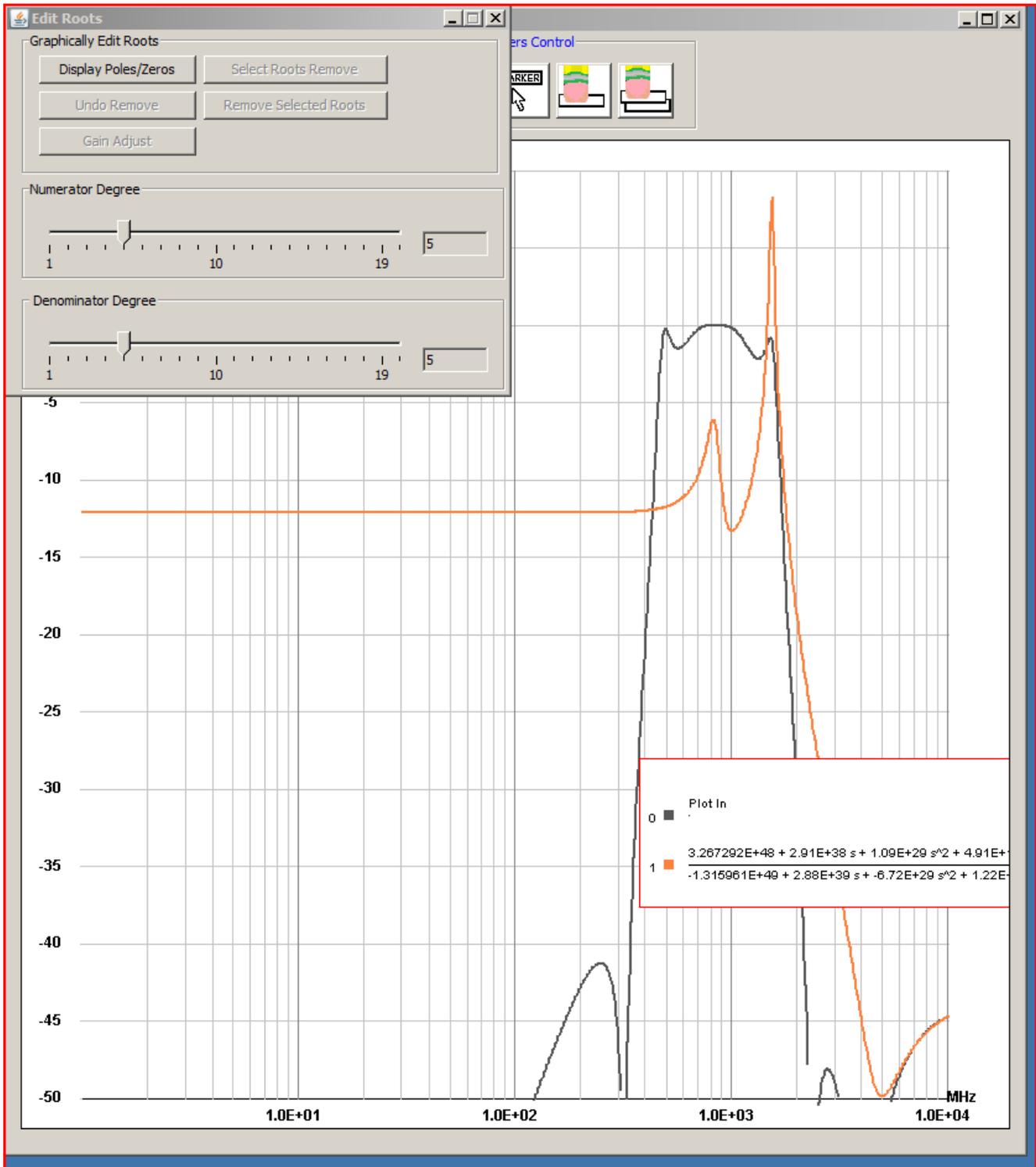
8. With a circuit of your choosing start the fitting procedure as shown in step 2 above.

The screenshot displays the Real Time Design 1.3.4 software interface. The main window shows a circuit diagram of a bandpass elliptical filter. The circuit includes a source (src) connected to a series resistor (I8, R = 50.0 Ohm), followed by a series inductor (I7, L = 1.49E+01 nH), and a series capacitor (I6, C = 2.27E+00 pF). The circuit then branches into two parallel paths. The first path consists of a series inductor (I5, L = 8.67E+00 nH, C = 3.89E+00 pF, R = 1.00E-02 mOhm) in series with a parallel combination of a capacitor (I4, C = 5.94E+00 nH, R = 7.70E-01 pF) and an inductor (I3, L = 4.39E+01 nH, R = 1.00E-02 mOhm). The second path consists of a parallel combination of an inductor (I2, L = 9.71E+00 nH, R = 1.00E-02 mOhm) and a capacitor (I1, C = 3.48E+00 pF, R = 1.00E-02 mOhm). The circuit is terminated by a series resistor (I1, R = 50.0 Ohm) connected to ground (IO).

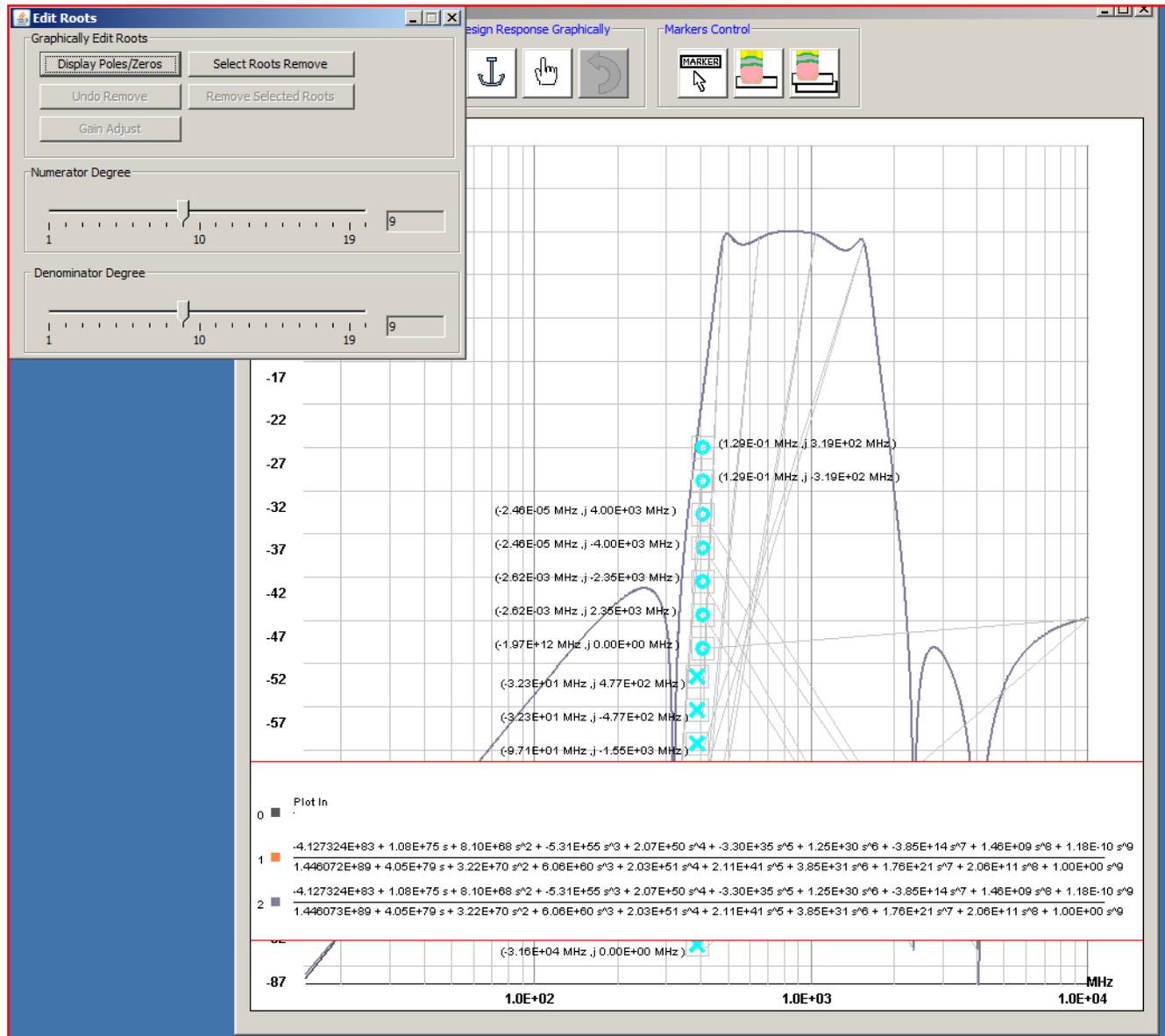
The interface includes a "Series Component" toolbar with icons for resistor, inductor, capacitor, and S2P block. The "Parallel Component" toolbar includes icons for inductor, capacitor, resistor, S2P block, and Z block. The "Filter Design" toolbar shows a filter synthesis icon.

The "Edit Component I7" panel is active, showing an "Inductor Adjust" slider set to 1.49E+01 nH. Below it, the "Edit Inductor Properties" panel shows Q = 1.000E10, at Frequency = 1.000E3 MHz, and Self Resonant Frequency = 1.000E14 MHz. The "Plot Waveform" section shows four plots for S11, S21, S21, and S21. The "Slider Adjust Course/Fine" and "S-Parameter Bisection" sections are also visible.

9. By adjusting the numerator and denominator degree slider the complex polynomial will best fit the data shown in the plot.



10. By pressing the “Display Poles/Zeros” button the poles and zeros will appear in the plot window.



11. While observing the pole/zero of interest (in this case zooming to the low frequency transmission zero), slide the slider of the circuit component to see the real time effect that changing it has on that particular root. While this is computationally expensive most machines have little trouble calculating this at speeds adequate for animation.

